# Chapter 2

# The Overlapping Generations Framework

## 2.1 An OG Model with Money

Here we shall conduct the same type of analysis as in Sections 1.1 and 1.2 but in an overlapping generations (OG) model. This is, as many readers will know, the second main type of framework for macro analysis of the dynamic general equilibrium type. In it, individuals do *not* live forever; they have finite lifetimes and do not leave bequests for their children. Thus at any point of time there are individuals alive who are different from each other—of different ages—even though everyone is alike at birth, so there is scope for certain transactions that do not occur in the Sidrauski model. For simplicity, we will assume that there are just two types—young and old persons. Thus each person lives for two periods and then dies. Accordingly, a period needs to be thought of as about 25 years, rather than as a year or a quarter.

Our first objective is to consider use of the model for monetary issues. A large fraction of the monetary analysis that has been done with OG models has assumed a special-case version in which money does *not* help to facilitate transactions. But in such models no one wants to hold money if there are other assets that yield a higher rate of return, and the result is a number of implications that are drastically inconsistent with reality. My contention is that in OG models, just as in infinite-lifetime models, there must be some feature of the setup that represents the transaction-facilitating properties of money. That is, there must be some way in which the model distinguishes between monetary (MOE) and non-monetary assets.

Consequently, my shortcut way of making an OG model suitable for monetary analysis is again to include real balances in the utility function. Most simply, suppose an individual born in t seeks to

$$\text{maximize } u(c_t, x_{t+1}, m_{t+1})$$

where $u_3 > 0$ for $m < m^*(x)$, and $u_3 \leq 0$ for $m \geq m^*(x)$, with $m^* > 0$. Here $c_t$ and $x_{t+1}$ are consumption in the individual's youth and old age, respectively. Of course $u_1 > 0$, $u_{11} < 0$, $u_2 > 0$, and $u_{22} < 0$.

Each individual has one unit of labor that he/she supplies inelastically when young (and none when old). When old, individuals produce by using capital they have saved together with hired labor of young people. The production function is $y_t = f(n_t,k_t)$, and is homogeneous of degree one. Note that the meaning of $n_t$ is the amount of labor *demanded* by a typical old person in t. We take $\nu$ to be the population growth rate. Money enters via lump-sum transfers to old people.

Under these assumptions, the budget constraints in period t for young and old people are

$$(2.1) \qquad w_t - c_t - k_{t+1} - \xi_t = 0$$

$$(2.2) \qquad f(n_t,k_t) + (1-\delta)k_t - w_t n_t + \frac{\xi_{t-1}P_{t-1}}{P_t} + tr_t - x_t = 0.$$

Here $\xi_t$ is real money balances held at the end of t, and we have the definition

$$(2.3) \qquad m_{t+1} = v_{t+1} + P_t\xi_t/P_{t+1}.$$

The lagrangian expression is

$$L_t = u(c_t x_{t+1}, \frac{\xi_t P_t}{P_{t+1}} + v_{t+1}) + \lambda_{1t}[w_t - c_t - k_{t+1} - \xi_t] +$$

$$\lambda_{2t}[f(n_{t+1},k_{t+1}) + (1-\delta)k_{t+1} - w_{t+1}n_{t+1} + \frac{\xi_t P_t}{P_{t+1}} + v_{t+1} - x_{t+1}].$$

Maximizing, we find

(2.4) $$\frac{\partial L_t}{\partial c_t} = u_1(c_t, x_{t+1}, m_{t+1}) - \lambda_{1t} = 0$$

(2.5) $$\frac{\partial L_t}{\partial x_{t+1}} = u_2(c_t, x_{t+1}, m_{t+1}) - \lambda_{2t} = 0$$

(2.6) $$\frac{\partial L_t}{\partial \xi_t} = u_3(c_t, x_{t+1}, m_{t+1}) \frac{P_t}{P_{t+1}} - \lambda_{1t} + \lambda_{2t} \frac{P_t}{P_{t+1}} = 0$$

(2.7) $$\frac{\partial L_t}{\partial k_{t+1}} = -\lambda_{1t} + \lambda_{2t}[f_2(n_{t+1}, k_{t+1}) + 1 - \delta] = 0$$

(2.8) $$\frac{\partial L_t}{\partial n_{t+1}} = \lambda_{2t}[f_1(n_{t+1}, k_{t+1}) - w_{t+1} = 0 .$$

Counting the budget constraints, the eight numbered conditions determine $c_t$, $x_{t+1}$, $m_{t+1}$, $\xi_t$, $k_{t+1}$, $n_{t+1}$, $\lambda_{1t}$, and $\lambda_{2t}$ for given values of $w_t$, $w_{t+1}$, $P_t$, $P_{t+1}$, and $tr_{t+1}$.

For competitive equilibrium (C.E.) we also need

(2.9) $$n_t = 1 + v,$$

which represents labor market clearing, and

(2.10) $$tr_t P_t = M_t - \frac{M_{t-1}}{1+v} ,$$

where $M_t \equiv$ money supply per old person in t after transfers, and

(2.11) $$\frac{M_t}{P_t} = \xi_t (1+v) ,$$

which equates the real money supply per old person in t to the money demand per young person multiplied by the number of young persons per old person. These eleven equations then determine paths for c, x, k, m, n, P, tr, $\xi$, $\lambda_t$, $\lambda_2$, and w *given* paths for M.

Note that since $m_t = tr_t + \frac{\xi_{t-1}P_{t-1}}{P_t}$ from (2.3), equation (2.10) gives

$m_t = \frac{M_t}{P_t} - \frac{M_{t-1}}{P_t(1+v)} + \frac{\xi_{t-1}P_{t-1}}{P_t}$. But then using (2.11) for $\xi_{t-1}$ we get

$$m_t = \frac{M_t}{P_t} - \frac{M_{t-1}}{P_t(1+v)} + \frac{M_{t-1}}{P_{t-1}(1+v)}\frac{P_{t-1}}{P_t} = \frac{M_t}{P_t}.$$

The last equality was derived, not assumed! Also, the foregoing equations together imply that

(2.12) $$f(n_t,k_t) + (1-\delta)k_t = x_t + (1+v)c_t + (1+v)k_{t+1},$$

which is the overall resource constraint. To get it, just put (1.30), (2.3) and (2.9) into (1.31), with the latter dated t. Then use (2.11) to get (2.12).

For a steady state we have

(2.13) $$f_1(1+v,k) = c + k + \frac{m}{1+v}$$

(2.14) $$f(1+v,k) = x + (1+v)c + (v+\delta)k$$

(2.15) $$\frac{u_1(c,x,m)}{u_2(c,x,m)} = f_2(1+v,k) + 1 - \delta$$

(2.16) $$[f_2(1+v,k)+1-\delta] = \left[\frac{u_3(c,x,m)}{u_2(c,x,m)} + 1\right]\frac{1}{1+\pi}.$$

These determine c, x, k, and m for a given, policy-determined $\pi$. That system is not recursive so superneutrality does not prevail.

But again there is an argument that suggests that there may be *near* super-neutrality. Suppose that $\frac{u_1(c,x,m)}{u_2(c,x,m)}$ is independent of m. Then equations (2.13) (2.14)

(2.15) would determine c, x, k if m did not appear in (2.13). But $k+\frac{m}{1+v}$ is the stock of wealth held by young people. In reality, m is only about 1 percent of real outside wealth (at least it is in the United States) so it is almost negligible relative to k. So near-superneutrality might be expected.

In the OG setup optimal inflation analysis is much trickier than with the Sidrauski model because there are different generations. But one can find conditions for Pareto optimality (P.O.) by maximizing utility of one generation subject to "given" values for all the others, or by assigning unspecified weights (which turn out not to matter) to all generations. In McCallum (1987) the approach is to maximize the welfare of the "initial period" old people. So we maximize $u(c_0, x_1, m_1)$ subject to

$$u(c_t,x_{t+1},m_{t+1})-u_t^* \geq 0,$$

for t = 1, 2, …, where the $u_t^*$ are arbitrary values, and also subject to the resource constraint (2.12) for each period. Let the Lagrangian be

$$Z_1 = u(c_0,x_1,m_1)+\theta_1[u(c_1,x_2,m_2)-u_t^*]+\theta_2[u(c_2,x_3,m_3)-u_3^*] + \ldots +$$

$$\varphi_1[f(1+\nu,k_1) + (1-\delta)k_1 - x_1 - (1-\nu)c_1 - (1-\nu)k_2]$$

$$+\ \varphi_2[f(1+v,k_2)+(1-\delta)k_2 - x_2 - (1+v)c_2 - (1+v)k_3] + \ldots.$$

Then

$$\frac{\partial Z_t}{\partial c_t} = \theta_t u_1(c_t,x_{t+1},m_{t+1})-(1+v)\varphi_t = 0 \qquad t = 1,2,\ldots$$

$$\frac{\partial Z_t}{\partial x_{t+1}} = \theta_t u_2(c_t,x_{t+1},m_{t+1})-\varphi_{t+1} = 0$$

$$\frac{\partial Z_t}{\partial m_{t+1}} = \theta_t u_3(c_t, x_{t+1}, m_{t+1}) = 0$$

$$\frac{\partial Z_t}{\partial k_{t+1}} = -\varphi_t(1+v) + \varphi_{t+1}[f_2(1+v, k_{t+1}) + 1 - \delta] = 0.$$

Also we have $u_2(c_0, x_1, m_1) - \theta_1 = 0$, $u_3(c_0, x_1, m_1) = 0$, and a transversality condition, namely,

(TC) $\lim_{t\to\infty} \varphi_t k_{t+1} = 0$.

From these we can eliminate the θ's and φ's to get

$$\frac{u_1(c_t, x_{t+1}, m_{t+1})}{u_2(c_t, x_{t+1}, m_{t+1})} = f_2(1+v, k_{t+1}) - \delta + 1$$

$$u(c_t, x_{t+1}, m_{t+1}) = u_t^*,$$

$$u_3(c_t, x_{t+1}, m_{t+1}) = 0,$$

for all t = 1, 2, …, plus the overall resource constraint and the TC. These four difference equations plus the TC determine paths for c, x, m, and k. There are "many" P.O. solutions (even for a given value of $k_1$) because there are many paths for the $u_t^*$.

Nevertheless, if we want to see if a particular C.E. path is P.O., we can use its values for $u(c_t, x_{t+1}, m_{t+1})$ as the $u^*_t$ values. Then the issue is just whether the other three difference equations and the T.C. are satisfied. In that regard, the resource constraint and $\frac{u_1}{u_2} = f_2 - \delta + 1$ *will* be satisfied, but $u_3 = 0$ will be satisfied by the C.E. only if $\frac{u_1(c_t, x_{t+1}, m_{t+1})}{u_2(c_t, x_{t+1}, m_{t+1})} = \frac{P_t}{P_{t+1}}$ – see e.g. (2.6) or (2.16). Thus P.O. requires that $(1+\pi)(f_2-\delta+1) = 1$ or $\pi + f_2 - \delta = 0$, where we neglect $\pi(f_2 - \delta)$. Thus the Chicago Rule is necessary for P.O. in the OG model. If it holds, P.O. will obtain if the TC is satisfied. [That is implied by our concavity assumptions, which make the FOCs plus TC sufficient (and the FOCs necessary) for an optimum.]

The relevant T.C. is $\lim_{t\to\infty} \varphi_t k_{t+1}=0$. My (1987) paper doesn't do a full analysis, but just assumes that the economy approaches a steady state.[1] Then $k_t$ approaches a constant so the TC depends on the limiting behavior of $\varphi_t$. And from $\frac{\partial Z}{\partial k_{t+1}} = 0$ we get

$$\varphi_{t+1} = \varphi_t \frac{1+v}{f_2(1+v,k_{t+1}) - \delta + 1}.$$

So in the vicinity of the steady state, $\varphi_t$ will shrink over time if $\frac{1+v}{f_2 - \delta + 1} < 1$. But that is the same as the condition from growth theory for the absence of capital over-accumulation—see Chapter 3. So if the Chicago Rule inflation rate is adopted, we will get P.O. unless there is capital over-accumulation.

In the rest of my 1987 paper, I argue that capital over-accumulation cannot occur in a model with land so long as the economy's production function leads it to approach a steady state. More generally, it cannot occur if the *land share* of income is bounded away from zero.

Thus analysis with an OG model leads to the same condition for the optimal average inflation rate as does the Sidrauski model, provided that the OG model is one that recognizes money's transaction-facilitating properties. Also, neither of these models implies superneutrality although both suggest that it may not be a bad approximation. So the two approaches are not actually very dissimilar in their policy-relevant implications, provided that the setup includes a shopping time, or MIUF, or CIA feature to reflect the

[1] That will happen in most cases but not in all.

MOE properties of money. It is well known (and will be shown below) that an OG model without any such feature leads to different and strange results—but so would an infinite-lifetime model without any such feature!

One way in which the models do differ is that the Sidrauski model has a steady-state marginal product of capital (MPK) that is independent of the inflation rate: $f_2 - \delta = v + \rho$. In the OG model, by contrast, we have

$$f_2 - \delta = \frac{u_1(c,x,m)}{u_2(c,x,m)} - 1$$

with c, k, m, and x all depending on $\pi$. This is an *advantage* for the OG model because it gives sensible results under the assumption that money growth is strongly negative. Suppose, that is, that $\mu = \pi < -\rho$ (assuming $v = 0$ and using $\mu$ to denote the growth rate of the aggregate money stock). Then the Sidrauski model suggests, neglecting cross-product terms, that $R = f_2 - \delta + \pi = \rho + \pi < 0$. But $R < 0$ is impossible. Thus the Sidrauski model's implications do not make sense for $\mu < -\rho$. But the OG model says that c/x will fall with decreases in $\pi$, which raises $f_2 - \delta$ and keeps $R \geq 0$.[2]

The models also differ in their implications for Ricardian equivalence. This topic will be explored in the problems at the end of this chapter.

Let me briefly comment on some writings mentioned in the bibliography below. The 1980 article by Wallace is one that argues very strongly in favor of doing monetary analysis in an OG model with *no* MIUF or shopping-time feature to reflect the transaction-facilitating services of money. The paper by Tobin (1980) argues that this is a very misleading way to do monetary analysis. My 1983 paper in the Carnegie-

[2] This implication is studied in McCallum (2000).

Rochester Conference Series argues that point again in a more analytical way and takes up several specific claims by Wallace. One of Wallace's complaints with MIUF or shopping-time models is that they do not explain *which* assets serve as money; but that complaint is not remedied by a model in which no asset serves as money. The 1990 *Handbook* article by Brock reviews this controversy, trying to find a compromise position (which is, I believe, impossible). The 1983 paper by Wallace reformulates his position in a different way, without talking at all about OG models. The interested reader might look at the introduction and conclusion to Wallace (1980), Blanchard and Fischer's Section 4.1, and Tobin's piece as a start. A summary of my arguments regarding monetary OG models appears, with some Sidrauski-model analysis, in my 1990 *Handbook* article. Some particular topics are taken up in Section 2.3 below.

## 2.2 Nonmonetary OG Analysis

OG models are quite well suited for several types of nonmonetary analysis, involving saving, social security systems, Ricardian equivalence, etc. Some of these issues are explored in Problems 1-4 and 8-9 at the end of this chapter.

## 2.3 OG "Monetary" Models With No MOE

In the foregoing OG model, one condition for individual optimality is that

$$\frac{\partial Z_t}{\partial \xi_t} = u_3(c_t, x_{t+1}, m_{t+1})\frac{P_t}{P_{t+1}} - \lambda_{1t} + \lambda_{2t}\frac{P_t}{P_{t+1}} = 0, \text{ or}$$

$$u_1(c_t, x_{t+1}, m_{t+1}) = [u_3(c_t, x_{t+1}, m_{t+1}) + u_2(c_t, x_{t+1}, m_{t+1})]\frac{P_t}{P_{t+1}}, \text{ or}$$

$$\frac{u_1(c_t, x_{t+1}, m_{t+1}}{u_2(c_t, x_{t+1}, m_{t+1}} = \left[\frac{u_3(c_t, x_{t+1}, m_{t+1})}{u_2(c_t, x_{t+1}, m_{t+1})} + 1\right]\frac{1}{1+\pi_t}, \text{ or}$$

$$(1+\pi_t)[f_2(n_{t+1},k_{t+1}) + 1 - \delta] = \frac{u_3(c_t,x_{t+1},m_{t+1})}{u_2(c_t,x_{t+1},m_{t+1})} + 1\,.$$

Approximately, then, we have

$$1 + \pi_t + f_2(n_{t+1},k_{t+1}) - \delta = \frac{u_3(c_t,x_{t+1},m_{t+1})}{u_2(c_t,x_{t+1},m_{t+1})} + 1$$

or

$$f_2 - \delta = \frac{u_3(c_t,x_{t+1},m_{t+1})}{u_2(c_t,x_{t+1},m_{t+1})} + (-\pi_t) :$$

| | | | | |
|---|---|---|---|---|
| net real rate of return on capital | = | marginal trans. services from holding money | + | real pecuniary return on holding money. |

In other words, the marginal rates of return on capital and money are the *same* when one takes account of the non-pecuniary return from holding money.

This type of relation will hold more generally. Suppose there are two different assets, both of which serve to facilitate transactions to some extent. But one of them pays interest between zero and $f_2 - \delta$, at (say) $\tilde{r}$. Then for that asset we will have (in equilibrium) a value of $u_3/u_2$ [the marginal non-pecuniary return] that is smaller than for zero-interest money. There may be a whole range of such assets that are partially money and partially interest bearing assets.

Indeed, in trying to develop an empirical measure of money, some people will use neither M1 nor M2 nor the monetary base but instead a weighted sum of various components, with the weights depending on the extent to which these items are used for transaction purposes. This extent is measured by their interest yields, relative to some short-term safe asset that is not used for transactions. This approach seems quite sensible.

In terms of the usual measures, it used to be more or less true that M1 was MOE money and the base was outside money (which is net wealth to the private sector). The former is no longer true because of the many new assets that have become available. But M2 never made any sense as a measure of MOE money—it became popular (I would guess) primarily because Friedman and Schwartz used it and they did so, I believe, largely because of the unavailability of M1 data before 1913. (That is not to suggest that they would agree with my imaginary reconstruction of their intellectual labors.)

Now let's see what happens in the OG model if we suppose that $u_3 \equiv 0$, i.e., that money does not provide transaction-facilitating services, regardless of the quantity held. In this case we cannot be certain that the chosen value of $\xi_t$ will be positive, so we have to use the two-part Kuhn-Tucker condition:

$$\frac{\partial Z_t}{\partial \xi_{t+1}} \leq 0 \text{ and } \xi_{t+1} \frac{\partial Z_t}{\partial \xi_{t+1}} = 0 .$$

We saw above that

$$\frac{\partial Z_t}{\partial \xi_{t+1}} = u_3(c_t, x_{t+1}, m_{t+1}) \frac{P_t}{P_{t+1}} - \lambda_{1t} + \lambda_{2t} \frac{P_t}{P_{t+1}}$$

and that $\lambda_{1t} = u_1(c_t, x_{t+1}, m_{t+1})$ with $\lambda_{2t} = u_2(c_t, x_{t+1}, m_{t+1})$ so now with $u_3 \equiv 0$ our two-part optimality condition is that

$$\left[ - u_1(c_t, x_{t+1}, m_{t+1}) + u_2(c_t, x_{t+1}, m_{t+1}) \frac{P_t}{P_{t+1}} \right] \leq 0$$

and

$$\xi_{t+1} \left[ -u_1(c_t, x_{t+1}, m_{t+1}) + u_2(c_t, x_{t+1}, m_{t+1}) \frac{P_t}{P_{t+1}} \right] = 0.$$

But we also have that $u_1(c_t,x_{t+1},m_{t+1}) = u_2(c_t,x_{t+1},m_{t+1})(f_2+1-\delta)$ so the expression

$$[-u_1(c_t,x_{t+1},m_{t+1}) + u_2(c_t,x_{t+1},m_{t+1})\frac{P_t}{P_{t+1}}]$$ becomes

$$-u_2(c_t,x_{t+1},m_{t+1})[f_2+1-\delta] + u_2(c_t,x_{t+1},m_{t+1})\frac{P_t}{P_{t+1}} = u_2(c_t,x_{t+1},m_{t+1})\left[\frac{P_t}{P_{t+1}} - (f_2-\delta+1)\right].$$

Thus the condition $\frac{\partial Z_t}{\partial \xi_{t+1}} \leq 0$ amounts to

$$\left[\frac{1}{1+\pi_{t+1}} - (f_2(n_{t+1},k_{t+1}) - \delta + 1)\right] \leq 0 \quad \text{or} \quad \frac{1}{1+\pi} \leq f_2 - \delta + 1 \quad \text{or} \quad 1 \leq (f_2 - \delta + 1)(1+\pi)$$

or to $0 \leq (f_2 - \delta + \pi)$. That is just saying that the nominal interest rate must be non-negative.

But suppose that we have a CE in which $R > 0$ or $\pi > -(f_2-\delta)$, which will be the case unless the inflation rate is unusually low. Then the strict inequality in the Kuhn-tucker condition will hold and individual optimality requires that $\xi_t = 0$. Thus *no money is demanded*. So when $u_3(c_t,x_{t+1},m_{t+1}) = 0$, the model implies that money will be valueless, the "price level will be infinite," unless $\pi \leq -$ MPK. If MPK, the marginal product of capital, is (e.g.) 3% p.a., money will be valueless unless we have an inflation of less than –3%.

But *of course* that is what this model would say with $u_3 \equiv 0$. Without doing any mathematical analysis at all, we can reason that if money provides no transaction-facilitating services, then it will not be held unless its pecuniary rate of return is as high as that provided by other assets. But its pecuniary real rate of return is $-\pi$. So $-\pi$ must be at least as large as $f_2 - \delta$ for money to be willingly held!

Empirically, this is preposterous. But it is the straightforward implication of the assumption that money provides no transaction-facilitating services. The problem is that *that* is a crazy assumption. It is as pointless to try to determine the value of money under that assumption as it would be to try to determine the value of a painting by Vermeer under the assumption that it provides no services to those persons who get to look at it [or to determine the value of a refrigerator assuming that it provides no food storage services].

Now let's briefly consider welfare analysis in this model with $u_3(c_t,x_{t+1},m_{t+1}) \equiv 0$. Well, *if* the inflation rate exceeds $-(f_2 - \delta)$ then money is not held and the model simply reduces to the Diamond (1965) OG model. In it, as implicit above on pp. 6-7, every C.E. is Pareto optimal unless the economy approaches a limiting condition with capital over-accumulation, i.e., with $f_2 - \delta < v$. In such cases, the limiting k exceeds the golden rule amount. If the inflation rate equals $-(f_2 - \delta)$, then some money may be held but P.O. still holds unless the system approaches over-accumulation.

What if the inflation rate is less than $-(f_2 - \delta)$? Well, it cannot be so in market equilibrium because that would make the pecuniary return on money higher than on capital so people would not willingly hold capital. Then as they try to reduce k, they raise $f_2$ and thus lower $-(f_2 - \delta)$ back to $\pi$. Formally, one condition of our model is that

$$\frac{u_3(c_t,x_{t+1},m_{t+1})}{u_2(c_t,x_{t+1},m_{t+1})}\frac{P_t}{P_{t+1}} + \frac{P_t}{P_{t+1}} = f_2(n_{t+1},k_{t+1}) - \delta + 1$$

so with $u_3(c_t,x_{t+1},m_{t+1}) \equiv 0$, $\frac{1}{1+\pi} = f_2 - \delta + 1$, in other words, $\pi \doteq -( f_2 - \delta)$.

Now the first-cited article by Wallace (1980) and Section 4.1 in Blanchard and Fischer conduct analysis in OG models with $u_3(c_t, x_{t+1}, m_{t+1}) \equiv 0$ and with a *constant* marginal product of capital: $f_2(n,k) - \delta = r$. In such models, money will not be held unless $\pi < -r$, so these models are useless vehicles for discussing monetary economics. Note also that they do not tell which assets function as money, they simply assume that none do. More generally, they assume that all included assets serve as MOE to the same extent. But without some distinction between monetary and nonmonetary assets, a model cannot be used for several central issues in monetary analysis.

One special case [due to Samuelson (1958)] has gotten a lot of attention, nevertheless. It is the model with $r = -1$, i.e., $1 + r = 0$, so that the good is completely perishable—storage is impossible. In that system there is no way for a person to save when he is young except to acquire the asset labelled "money." So if the endowments are large for young people, who then want to save for old age, people will acquire money when young and trade it for goods when old. To do so and thereby adjust their lifetime consumption pattern is likely to be Pareto-superior to being unable to save. But all this tells us is that it is useful—indeed, important—for an economy to have *some* store of value. If an economy has no store of value, people's utility will (usually) be harmed. This tells us nothing about money. Indeed, one should not call this paper asset in this model "money." It would be better to think of it as a government bond, one that pays no explicit interest. Its real rate of return will of course[3] be $\nu - \mu$ where $\mu$ is the rate of creation of these bonds. Some people call them money and call $\mu - \nu$ the "inflation rate." [See, e.g., Grandmont (1985, p. 1031), Blanchard and Fischer, p. 161.] But that is seriously misleading because it suggests that the monetary authority in an actual economy

[3] Why "of course?" Well, in a steady state M/P will be constant, where M is the stock of these bonds per young person. So P will grow at the same rate $\pi$ as M, which is $\mu - \nu$ when $\mu$ is the aggregate growth rate of bonds. With no interest paid, the real pecuniary rate of return is $-\pi$ which equals $-(\mu-\nu)$.

will lower the real rate of interest point-for-point if it increases the rate of money growth. But almost no one believes that to be true in reality. And both of our models with transaction-facilitating money suggest that near-superneutrality holds, i.e., that r is hardly affected (in steady state equilibrium) by the rate of money growth.[4]

In fact, Blanchard and Fischer finally agree that the OG model with $u_3 \equiv 0$ is a crazy model to use for monetary analysis—see their p. 164. But they say this so blandly and devote so much space to these models (pp. 156-164) and mention them so often in other places that I think their presentation has a seriously misleading perspective—is virtually twisted out of shape. A highly personal review of their Chapter 4 follows, as an appendix.

[4] This belief is supported by a quantitative calibration exercise in McCallum (2000).

## Appendix—A Review of Chapter 4 of Blanchard and Fischer (1989)

In their Section 4.1 Blanchard and Fischer analyze the OG model in which money does not serve as a MOE. They obtain many striking "results," but at the end recognize that this is an unsatisfactory model for studying money. (I would agree but would also say that the discussion has nothing to do with money, it is about saving.)

In 4.2 (which is good but very short) they argue that transactions may be facilitated by use of money, or by a credit system. Money will be better for some transactions. They mention the cash-in advance constraint (but not other transaction setups) and then the precautionary demand for money. These they decide are too rigid.

In 4.3 they describe an OG model with a transaction technology that is of the Baumol-Tobin type. (The model is David Romer's.) This is a sensible model but their analysis with it includes a serious mistake—they say that the Chicago Rule does not hold when the steady state MPK is positive. We have emphasized that point above.

In 4.4 they ask whether one-time changes in M, which will be neutral from a long-run perspective, will have short-run (transitory) effects on real variables. The setup they use is one in which some agents are by assumption prevented, after a shock occurs, from exchanging bonds for money until the next period. But that length of time in actual economies would be a few hours at most, so this setup is inappropriate for talking about non-neutralities that show up importantly in quarterly and annual data (i.e., business cycles). At the end they recognize this—so why did they introduce the model at all?

In 4.5 they analyze the Sidrauski model. That is fine, but at the end they get the welfare analysis wrong, as we have seen.

Section 4.6 is only two pages on a number of topics involving the distinction between inside and outside money. It is a good and sensible discussion.

Section 4.7 uses "an even more drastic shortcut" than "putting money in the utility function" namely, "to start by directly specifying the demand for money function." They use the Cagan demand function to discuss government revenue from creating money. Parts of this I like, parts I don't. Why do they use adaptive expectations instead of rational expectations? But I will also use the Cagan demand function in a later section of this course. My justification is provided by the discussion leading to equation (1.16) of Section 1.1, plus the assumption that the market-clearing values of $y_t$ and $r_t$ are (approximately) constant over time.

The foregoing comments have emphasized aspects of Blanchard and Fischer's Chapter 4 that, in my opinion, need qualification or correction. It should therefore be added explicitly that their text is, overall, an impressively rich and wide-ranging piece of work, from which I have certainly learned much myself.

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## Problems

1. Obtain a version of Peter Diamond's famous1965 OG model by eliminating monetary variables from the model given on pages 2-4. Derive equations that determine competitive equilibrium paths for $c_t$, $x_t$, $k_t$, $w_t$, and $n_t$.

2. Consider a special case of this model in which

$$u(c_t, x_{t+1}) = \theta \log c_t + (1-\theta) \log x_{t+1}$$

$$f(n_t, k_t) = A n_t^{1-\alpha} k_t^{\alpha}$$

where $0 \leq \theta, \alpha, \delta \leq 1$. Derive a competitive equilibrium solution expression for $k_{t+1}$ in terms of $k_t$. Is the implied equilibrium path stable? What is the equilibrium steady state value of the marginal product of capital?

3. Generalize the model of problem 1 by adding government bonds and lump-sum taxes on young people. (Let $b_{t+1}$ = real bonds purchased in t by a young person, who pays taxes of $-tr_t$.) Does Ricardian equivalence prevail?

4. Consider again the Diamond model as specified in Problem 1. Does its competitive equilibrium satisfy conditions for Pareto optimality with respect to the current old (as of period 1) and persons born in periods t = 1, 2 …? (Here t = 1 is the date at which the Pareto optimality problem is being solved.

5. Suppose that initially a monetary OG economy is in a steady state with v = 0, $f_2(1,k) - \delta = 0.64$, and $\mu = 0$. ($\mu$ is the money stock growth rate.) Now suppose that $\mu$ is changed to –0.80 and kept at that value for many periods—enough that a new steady state is (virtually) attained. What will happen to the nominal and real rates of interest? To the ratio c/x?

6. What can be said regarding social optimality in the initial and subsequent steady states in problem 5?

7. What is the numerical dollar value of net money holdings by the private sector of the U.S. economy? What is the value of this sector's net non-monetary assets? (In both cases, "net" means after the sector's debts are cancelled against its assets.)

8. Add a "pay-as-you-go" social security system (i.e., a compulsory national pension system) to the model of problem 1, as follows. Let $ssy_t$ be a lump sum tax imposed on each young person in period t, and let $sso_t$ be the (lump sum) payment to each old person in t. Then a pay-as-you-go scheme is expressed by the system's budget constraint $sso_t = (1+\nu)ssy_t$. Determine whether the presence of such a system increases or decreases the economy's steady state stock of capital per person.

9. Redo problem 8, now assuming that the social security system is "fully funded." That is, each young person in t is required to contribute $ssy_t$ to a fund that is invested at the prevailing rate of interest, $r_t \equiv f_2(n_{t+1}, k_{t+1} + ssy_t) - \delta$. (In other words, the system purchases goods in t and then in t+1 loans them to producers who return them later in t+1 with interest.) Then when old a person born in t receives a payment of $sso_{t+1}$ from the system, while the system's operating rule governing this payment is $sso_{t+1} = (1+r_t)ssy_t$. (Except for administrative expenses, here neglected, this system would be equivalent to requiring each young person to invest the amount $ssy_t$ in capital.)