The Fiscal Theory of the Price Level is Not Learnable

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1. Introduction

During the past several years, a striking body of literature has appeared in which it is argued that general price level determination is essentially a fiscal, rather than monetary, phenomenon. The most prominent papers have been those of Woodford (1994, 1995, 2001), Sims (1994, 1997), Leeper (1991), and Cochrane (1998, 2000), but there are several others of significance.[1] If the theory expounded in these papers was valid empirically, there would be major implications for the manner in which fiscal and monetary policies should be conducted, not only in individual economies but also within monetary unions.[2] The purpose of the present paper is to describe this theory and provide one major reason why I believe that it is not relevant to actual economies, but instead is basically misleading. That position has been put forth in McCallum (2001a), but the present argument includes a new, different, and more satisfactory justification. In addition, the basic exposition is simplified here and several extensions are developed.

At the outset it should be emphasized just how drastically unorthodox or counter-traditional the fiscal theory of price level determination is.[3] Specifically, it does not merely suggest that fiscal as well as monetary policy stances are significant for price level behavior; instead it features a case in which only fiscal policy is relevant. In the prototype analysis presented below, the price level moves over time in a manner that mimics the path of government bonds outstanding and is entirely unlike the path of the stock of high-powered money. Accordingly, it is clearly not the case that the argument

[1] An incomplete list would include Carlstorm and Fuerst (2001), Christiano and Fitzgerald (2000), Kocherlakota and Phelan (1999), and Schmitt-Grohe and Uribe (2000). Some of these are not entirely supportive of the fiscal theory, whereas Buiter (1998, 1999) is entirely non-supportive.
[2] On this matter, see Bergin (2000), Dupor (2000), and Sims (1997).
[3] In what follows, I shall for brevity often refer to the latter as the "fiscal" or "fiscalist" theory.

involves fiscal behavior that drives an accommodative monetary authority, as when rapid base money growth is adopted to finance a fiscal deficit.[4] Indeed, it is this drastic aspect of the fiscal theory that has made it a subject of great interest.[5] In this regard, an important point is that the type of model typically utilized in the literature's analysis is not of the overlapping generations type, in which the Ricardian equivalence proposition is known to fail—implying that tax changes will affect price level behavior. Instead, the model is basically of the Sidrauski-Brock type, in which Ricardian equivalence results are normally obtained, i.e., results implying that bond-financed tax changes have no effect on the price level or other macroeconomic variables of primary interest.[6] In such a setting, fiscalist positions are truly startling.

2. Basic Formulation

As a background for illustrating these drastic results, let us begin with an orthodox analysis of price level determination in an extremely simple and transparent setting.[7] Suppose that the (per capita) money demand function for a closed economy is of the textbook form

$$(1) \qquad m_t - p_t = c_0 + c_1 y_t + c_2 R_t + v_t \qquad\qquad c_1 > 0,\ c_2 < 0,$$

where m_t, p_t, and y_t are logs of the (base) money stock, price level, and output (income) for period t, while R_t denotes a one-period nominal interest rate. The disturbance v_t is taken for simplicity to be white noise. It is well known that there are rigorous dynamic general equilibrium models with optimizing agents that will justify (1) as a linear

[4] Thus the theory is fundamentally different from that of Sargent and Wallace (1981).
[5] Woodford (1995, pp. 25-26), too, has emphasized the importance of cases in which monetary and fiscal impulses diverge so that "it is possible to see which is truly determinative." Also see Buiter (1999) and Cochrane (1998). Important points about the nature of traditional or monetarist views are made by Nelson (2002).
[6] For an analysis based on this Ricardian model, see McCallum (1984).
[7] This section is adapted from McCallum (1999c).

approximation to a combination of implied Euler equations (first-order optimality conditions).[8] The present exposition is intended to convey the essential features of a full optimizing analysis while ignoring some of the details.[9]

Furthermore, let us assume that the economy is one in which output and the real rate of interest are constant over time. Then (1) collapses to

(2) $m_t - p_t = \gamma + \alpha\,(E_t p_{t+1} - p_t) + v_t \qquad \alpha = c_2,$

which is the familiar Cagan specification for money demand. And let us for now consider cases in which the growth rate of the (base) money stock is kept constant by the central bank, so that

(3) $m_t = m_{t-1} + \mu,$

where μ is the growth rate of the money stock. These relations plus rational expectations determine the behavior of p_t and m_t for time periods t = 1, 2, …. It is possible that the structure was different prior to period 1.

In this setting, the orthodox bubble-free or "fundamentals" rational expectations (RE) solution for p_t can be found by conjecturing that it is of the form

(4) $p_t = \phi_0 + \phi_1 m_{t-1} + \phi_2 v_t,$

since m_{t-1} and v_t are evidently the system's only state variables. In that case we have $E_t p_{t+1} = \phi_0 + \phi_1(m_{t-1} + \mu)$ so substitution of the latter, (3), and (4) into (2) yields

(5) $m_{t-1} + \mu = \gamma + \alpha\,[\phi_0 + \phi_1(m_{t-1} + \mu)] + (1\text{-}\alpha)\,[\phi_0 + \phi_1 m_{t-1} + \phi_2 v_t] + v_t.$

The latter implies that for (4) to be a solution, i.e., to hold for all realizations of v_t and m_{t-1}, we must have satisfaction of the undetermined-coefficient (UC) conditions

[8] See, for example, Woodford (1995, 2001), Sims (1994), McCallum (1999a, 2001a), Kocherlakota and Phelan (1999), and others.

(6) $1 = \alpha\phi_1 + (1-\alpha)\,\phi_1$

$0 = (1-\alpha)\,\phi_2 + 1.$

$\mu = \gamma + \alpha\phi_1\mu + (1-\alpha)\,\phi_0 + \alpha\phi_0.$

Thus we have that $\phi_1 = 1$, $\phi_2 = -\,1/(1-\alpha)$ and $\phi_0 = \mu - \gamma - \alpha\mu$, i.e., that the solution is

(7) $p_t = \mu(1\text{-}\alpha) - \gamma + m_{t-1} - [1/(1-\alpha)]v_t = m_t - (\gamma + \alpha\mu) - [1/(1-\alpha)]v_t.$

Here we see that p_t grows one-for-one with m_t, i.e., the price level P_t moves on average in proportion to the money stock M_t, but fluctuates around this average position in response to realizations of v_t. Specifically, p_t is temporarily reduced by positive money demand shocks ($v_t > 0$) or boosted by negative shocks ($v_t < 0$). This is clearly an entirely traditional—one might even say "monetarist"—analysis of price level behavior in the economy in question.

For an even simpler special case, let us next suppose that the money growth rate is zero, i.e., that $\mu = 0$ so that $m_t = m$. Then the solution for p_t is

(8) $p_t = m - \gamma - v_t/(1-\alpha).$

Thus, if money demand shocks were absent we would have $p_t = m - \gamma$.

It must be noted, however, that while (7) and its special case (8) give the well-behaved, orthodox, bubble-free RE solutions for this model, there are other expressions as well that satisfy the model (2)(3) with RE. For simplicity, let us consider the special case with constant $m_t = m$, but now conjecture a solution of the form

(9) $p_t = \psi_0 + \psi_1\, p_{t-1} + \psi_2 v_t + \psi_3 v_{t-1},$

instead of $p_t = \phi_0 + \phi_2 v_t$. Then working through the same type of analysis as before, one finds that the UC conditions analogous to (6) are

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[9] Very recently, Evans and Honkapohja (2002b) have conducted a similar analysis in a fully optimizing

$$(10) \quad 0 = \alpha\, \psi_1^2 + (1-\alpha)\, \psi_1$$

$$0 = \alpha\, \psi_1\, \psi_2 + \alpha\, \psi_3 + (1-\alpha)\, \psi_2 + 1$$

$$0 = \alpha\, \psi_1\, \psi_3 + (1-\alpha)\, \psi_3$$

$$m = \gamma + \alpha\, \psi_0 + \alpha\, \psi_1\, \psi_0 + (1-\alpha)\, \psi_0.$$

By inspection, therefore, we see that the first of these has two roots $\psi_1^{(1)} = 0$ and $\psi_1^{(2)} = (\alpha-1)/\alpha$. If the former is the relevant root, then we find that $\psi_3 = 0$, $\psi_2 = -1/(1-\alpha)$, and $\psi_0 = m - \gamma$ so that the same solution as in (8) is obtained. But if $\psi_1^{(2)}$ is relevant, then $\psi_3 = -1/\alpha$ and $\psi_0 = (m - \gamma)/\alpha$ while <u>any</u> value of ψ_2 is possible. So an infinity of solution paths is in this case consistent with the model. Note, moreover, that $\psi_1^{(2)} = (\alpha-1)/\alpha > 1.0$, so most of these solution paths are explosive. One such path is illustrated in Figure 1, where the random component is suppressed.

Of course there are other variables and conditions besides those discussed thus far in a fully articulated model of the economy under consideration. In particular, let B_{t+1} denote the quantity of one-period government bonds purchased in t, with each bond purchased at the price $1/(1+R_t)$ and redeemed in t+1 for one unit of money. Then it is typically true that a fully-specified optimizing analysis would require that

$$(11) \quad \lim_{j\to\infty} E_t\, \beta^j\, (M_{t+j} + B_{t+j})\,/P_{t+j} = 0,$$

i.e., that a transversality condition pertaining to real financial wealth must be satisfied. Here β is a typical agent's discount factor, $\beta = 1/(1+\rho)$, with $\rho > 0$ so that $0 < \beta < 1$.

We are now at last prepared to turn to the fiscalist theory. With government

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setup. Their basic result agrees with the one reported here. Other results of theirs are mentioned below.

bonds recognized, we can write the consolidated[10] government budget constraint (GBC) in per capita terms as

(12) $P_t (g_t - tx_t) = M_{t+1} - M_t + (1 + R_t)^{-1} B_{t+1} - B_t,$

where g_t and tx_t are real government purchases and (lump sum) tax collections, respectively. In real terms, this constraint could then be expressed as

(13) $g_t - tx_t = (M_{t+1} - M_t)/P_t + (1 + R_t)^{-1} (P_{t+1}/P_t) b_{t+1} - b_t,$

where $b_t = B_t/P_t$. The reader should please note the mixed notation being utilized: $b_t = B_t/P_t$ whereas $m_t = \log M_t$ and $p_t = \log P_t$. Condition (13) obtains for t = 1, 2, ….

Now consider the special case of the economy discussed above in which M_t and thus m_t are constant. Also let the random shock v_t be absent so that P_{t+1} is correctly anticipated in t and suppose that fiscal policy aims for a constant surplus $tx_t - g_t = s > 0$ with $g_t = g$. Then with the real rate of interest on bonds r_t defined by $1 + r_t = (1 + R_t)/(1 + \pi_t)$, where $\pi_t = (P_{t+1} - P_t)/P_t$, and with $r_t = \rho$, as would be implied by optimizing behavior in the absence of shocks,[11] the government budget constraint becomes

(14) $b_{t+1} = (1 + \rho) b_t + (1 + \rho) (g_t - tx_t) \qquad t = 1,2,\ldots.$

But since $1 + \rho > 1.0$, if $g_t - tx_t$ is constant the last equation reveals a strong tendency for b_t to explode as time passes. As t grows without limit, b_t approaches growth at the rate ρ, i.e., behaves like $(1+\rho)^t$. Thus the transversality condition (11) tends to be violated since growth of b_t just offsets the shrinkage of $\beta^t = 1/(1 + \rho)^t$, yielding a limit that is a positive constant.

In fact, in this case there are just two paths for b_t that, with $g_t - tx$ constant, will satisfy (14) and also (9)(10)(11) for t = 1,2,…. One of these obtains if the value b_1 equals

[10] The government consists of a fiscal authority and a central bank.

$-(1 + \rho)(g - tx)/\rho$, for then (14) implies that

$$(15) \quad b_2 = (1 + \rho)[-(1 + \rho)(g - tx)/\rho] + (1 + \rho)(g - tx)$$

$$= (1 + \rho)(g - tx)[-(1+\rho)/\rho + 1] = -(1 + \rho)(g - tx)/\rho$$

and that same value prevails in all succeeding periods. But $b_1 = B_1/P_1$, and B_1 is the number of nominal government bonds outstanding at the beginning of the initial period, t = 1. Thus if the price level in this first period, P_1, adjusts to equal the value $P_1 = B_1\rho/(1 + \rho)(tx - g)$, then condition (11) as well as (14) will be satisfied. Indeed, this is precisely what the fiscalist theory predicts: that P_1 adjusts relative to B_1 and $tx - g > 0$ so as to satisfy the individual agents' optimality condition (11).[12]

But what about the necessary condition for money holdings, equation (2)? Here the fiscalist answer is that although the path just described will not conform to the $p_t = m - \gamma$ fundamentals solution implied by (8), it can and will satisfy the alternative solution $p_t = [(\alpha - 1)/\alpha]p_{t-1} + (m - \gamma)/\alpha$ for all t = 2,3,….[13] The price level P_1, and thus p_1, is determined by B_1 and the value of b_1 necessary to satisfy (11), with subsequent P_t and p_t values being given by (9) with $\psi_1 = (\alpha - 1)/\alpha$. The price level explodes as time passes, despite the constant value of M_t, but all of the model's equilibrium conditions including RE are satisfied nevertheless. Since P_t and B_t are growing at the same (explosive) rate, while M_t is constant, the outcome is rightfully regarded as highly "fiscalist."[14]

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11 See, e.g., McCallum (1999a, 2001a), Kocherlakota and Phelan (1999), or Woodford (1995)..

12 Note that with b_t constant but positive, the government's real revenues from bond sales are negative ($-b\rho/(1+\rho)$).

13 It might be asked why this relation does not determine p_1 in relation to p_0. I believe that the answer is that, in the full stochastic model, it determines the value of ψ_2 in (9).

14 There is a serious problem, however, with this solution if B_1 is such that the implied value of P_1 is smaller than $P^* = Me^{-\gamma}$. In this case the fiscalist equilibrium does not exist because P_t approaches 0 leading to violation of the transversality condition (11). Also, if $tx - g < 0$, then a negative price level would be required for satisfaction of (14) by the assumed value of b_1. This problem is stressed by Buiter (1998, p. 20).

Now let us consider the one other path of b_t that will, with g – tx constant, satisfy the TC (11) as well as (9), (10), and (14). It is that $b_{t+1} = 0$ for all t = 1, 2 ,…. Clearly, (11) will be satisfied with $B_{t+1} = 0$ and in that case places no constraint on P_t values. Thus these are free to obey $p_t = m - \gamma$, as in the special case of (9)-(10) given by (8). Therefore this solution is the orthodox or monetarist solution.

It might be asked how the GBC (14) can be satisfied with this solution, i.e., with $B_{t+1} = 0$ for t = 1, 2, … and $tx_t - g > 0$. The explanation is as follows. In a market economy, it is not legitimate to specify fiscal policy as controlling both g_t and tx_t (with an M_t path given) because such a policy could imply that bonds sold to the private sector are greater than the demand for them. Thus we need to distinguish between bond supply B^S_{t+1} and bond demand B^D_{t+1}, and policy is appropriately specified in terms of M_t, g_t, and B^S_{T+1} with the relevant equilibrium condition being $B^D_{t+1} \leq B^S_{t+1}$. In the case at hand, the planned value of $tx_t - g > 0$ reflects B^S_{t+1} plans, whereas the realized values involve $B_{t+1} = B^D_{T+1} = 0$ and $tx_t - g = 0$. The tx – g values realized are smaller than planned because real revenues from bond sales are larger—zero, rather than the planned negative value. It is not surprising that some such adjustment is needed since the experiment at hand has monetary (M_t) and fiscal (g_t and B^S_{t+1}) policies being set independently and exogenously. The monetarist and fiscalist solutions reflect two different ways by which these potentially conflicting policies can be reconciled.[15]

In sum, we end up with two RE solutions that represent two competing hypotheses regarding price level behavior in the hypothetical economy under study. It is

[15] For more discussion of this topic, see McCallum (2001a). For an alternative formalization, see Kocherlakota and Phelan (1999). A substantively different and perhaps more extreme position is that of

an economy in which the money stock is constant over time, all behavioral relations are constant, and there are no stochastic disturbances impinging upon its agents or productive processes. According to the monetarist hypothesis, the price level is constant through time at a value that is proportional to the magnitude of the money stock, and no government bonds are purchased by private agents.[16] By contrast, the fiscalist hypothesis implies that, despite a constant money stock, the bond stock and the price level both explode as time passes—but without violating any optimality condition for private agents. This happens because the initial price level adjusts relative to the initial bond stock so as to make the real bond stock equal the single non-zero value that will permit the stock of real bonds to remain constant and the transversality condition (11) to be satisfied. Under this latter hypothesis, the initial price level is proportional to the initial bond stock and the price level grows in tandem with the bond stock.

The crucial issue is, which of the two solutions provides the better guide to reality, i.e., to price level behavior in actual economies? In previous writings (McCallum 1999a, 2001a) I have emphasized that the traditional equilibrium is the "fundamentals" or "bubble-free" solution provided by the minimum-state-variable (MSV) solution concept for RE models, whereas the fiscalist solution represents a bubble solution.[17] I have suggested that this is a plausible reason—in addition to the empirical evidence—for preferring the former, but it must be recognized that for many analysts that argument may not be persuasive.[18] Accordingly, the next section of the present paper will develop a

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Buiter (1998, p. 17), who argues that the fiscalist assumptions "violate the normal rules for constructing a well-posed general equilibrium model."

[16] This does not imply that none are offered for sale by the government.

[17] The MSV solution concept is extensively discussed by McCallum (1999b). It is crucial to recognize that in linear models the MSV solution is, by definition/construction, unique.

[18] For example, Woodford (2001, p. 701) argues that "what constitutes a 'bubble equilibrium' is often in the eye of the beholder…."

more substantive theoretical reason to view the traditional MSV solution as economically relevant, and the fiscalist solution as irrelevant. This reason is based on the intimately-related concepts of E-stability and least-squares learnability.

3. E-Stability and Learnability

Iterative E-stability was developed in the 1980s, principally by Evans (1985, 1986), and then modified in response to work by Marcet and Sargent (1989). Iterative E-stability involves a thought experiment in which one conceives of expectational behavior with anticipated variables such as p^e_{t+1} being described by an expression of a form that would be appropriate under RE, but with parameter values that are initially incorrect.[19] This "expectation function" implies, when substituted into the model of the economy, a law of motion that entails systematic expectational errors. So one can then conceive of revised values of the parameters of the expectation function that are suggested by the law of motion. These too will imply incorrect forecasts, but one can imagine continuing with a series of iterations and consider whether they will converge to a specific RE solution, be it the MSV or a non-MSV solution.[20] If such a process converges to a particular solution, then the latter is said to be iteratively E-stable.

To illustrate the concept of iterative E-stability, let us consider an example that is similar to the model of Section 2 with $m_t = m$. Thus suppose that some unspecified variable y_t is generated by the structural model

(16) $y_t = a_0 + a_1 E_t y_{t+1} + u_t$,

where $u_t = \xi u_{t-1} + \varepsilon_t$ with $|\xi| < 1$ and ε_t being white noise. With this specification, the usual "fundamentals" RE solution will be of the form

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[19] Here p^e_{t+1} denotes the subjective expectation of p_{t+1} formed at time t, not necessarily according to RE.
[20] If there is convergence, it will be to some RE solution.

(17) $y_t = \phi_0 + \phi_1 u_t$,

but suppose that agents do not "initially" know the exact values of the ϕ_j parameters. If at any date t the agents' prevailing belief is that their values are $\phi_0(n)$ and $\phi_1(n)$, then the <u>perceived law of motion</u> (PLM) will be[21]

(18) $y_t = \phi_0(n) + \phi_1(n)u_t$,

and the implied expectation of y_{t+1} will be

(19) $\phi_0(n) + \phi_1(n)\xi u_t$.

But using this expression in place of $E_t y_{t+1}$ in (16)—which implies temporarily abandoning RE—gives

(20) $y_t = a_0 + a_1[\phi_0(n) + \phi_1(n)\xi u_t] + u_t$

as the system's <u>actual law of motion</u> (ALM). Now imagine a sequence of iterations from the PLM to the ALM. Writing the left-hand side of (20) in the form (18) for iteration n+1 gives $\phi_0(n+1) + \phi_1(n+1)u_t = a_0 + a_1[\phi_0(n) + \phi_1(n)\xi u_t] + u_t$ and therefore implies that

(21a) $\phi_0(n+1) = a_0 + a_1\phi_0(n)$

(21b) $\phi_1(n+1) = a_1\phi_1(n)\xi + 1$.

The issue, then, is whether iterations defined by (21) are such that the $\phi_j(n)$ converge to the ϕ_j values in (17) as $n \rightarrow \infty$. For this simple example, it can be seen by inspection that necessary and sufficient conditions for such convergence are $|a_1| < 1$ and $|\xi a_1| < 1$. If these conditions hold, then the solution (17) is said to be iteratively E-stable. Evans (1986) found that in several prominent and controversial examples the MSV solution is iteratively E-stable.

[21] Here n is being used to index iterations in an eductive process of learning in meta-time.

By considering ever smaller "time periods" for iterations of the foregoing type one can develop a related process that is continuous in notional time (meta-time).[22] Evans and Honkapohja (1999, 2001) emphasize this unqualified notion of E-stability because it is, under fairly general conditions, equivalent to learnability in actual time by means of a least-squares-based adaptive process.[23] Absence of E-stability therefore implies that a particular RE solution will not be obtained if agents are not endowed with knowledge of the model's true parameters, but attempt to learn them by statistical estimation based on data generated to date, with updating and re-estimation taking place each period. Of course this conclusion presumes that the statistical processing at each date is of the least-squares type, but that is a distinctly reasonable way for the agents to proceed; the general idea is that if this process does not permit agents to acquire knowledge of the true parameter values then they are unlikely to do so by other processes. Technical results pertaining to the near-equivalence between E-stability and least-squares learnability are described in detail by Evans and Honkapohja (1999, 2001).

4. Application to Fiscal Theory

Here our objective is to develop E-stability results for the model of Section 2 with $m_t = m$. Specifically, we want to determine whether either, or both, of the solutions given by equations (10) features E-stability and thus least-squares learnability. We can make a start by writing the model in question in as

(22) $p_t = [\alpha/(\alpha-1)]\, E_t p_{t+1} + (m-\gamma)/(1-\alpha) + [1/(\alpha-1)]v_t,$

which is of the same form as that examined in Section 3 with the parameter a_1 in (16) equal to $\alpha/(\alpha-1)$ and $v_t = \xi v_{t-1}$ + white noise. Therefore, with $\alpha < 0$ we have $0 < a_1 < 1$, so

[22] Evans (1989) adopted the unqualified concept following results described in Marcet and Sargent (1988, 1989).

for any ξ such that $| \xi | < 1$, the MSV solution based on $\psi_1^{(1)} = 0$ in (10) is iteratively E-stable. That does not establish, however, that the solution based on $\psi_1^{(2)} = (\alpha-1)/\alpha$ is not E-stable.

Fortunately, however, this model is a particular version of one case studied by Evans (1986), which shows that with $0 < a_1 < 1$, the MSV solution based on $\psi_1^{(1)}$ is recursively E-stable whereas the non-MSV solutions involving $\psi_1^{(2)} = (\alpha-1)/\alpha$ are not recursively E-stable (1986, p. 153) under parameter values that include ours. These results were shown to hold for unqualified E-stability, moreover, by Evans (1989, pp. 311-312). Recently, moreover, a very thorough analysis of that model was provided by Evans and Honkapohja (2002a), who consider still more solutions—ones of the "resonant frequency sunspot" type. Their finding is that equilibria reflecting this latter type of phenomena will be E-stable only if the coefficient analogous to $\alpha/(\alpha-1)$ in (22) satisfies $\alpha/(\alpha-1) < -1$. But with $\alpha < 0$, that is not possible in the model at hand.

It remains to be settled whether this case is one for which E-stability goes hand in hand with least-squares learnability. The basic result, established by Evans and Honkapohja (2001, pp. 231-235), is that E-stable solutions that are dynamically stable are learnable, whereas E-unstable solutions are not. Therefore, the traditional monetarist solution is, and the fiscalist solution is not, learnable by adaptive least-squares procedures in the model of Section 2. This is the basic result of the present paper.[24]

5. Generalization of Basic Result

Ignoring stochastic terms, the linear model that we have used to this point can be

[23] The link is tighter when the solution in question is dynamically stable, i.e., non-explosive.

[24] More recently, Evans and Honkapohja (2002b) have considered the fiscal-theory issue specifically in a fully optimizing, nonlinear formulation. Their conclusion for the basic case corresponding to the one of

represented graphically as in Figure 1. There the traditional MSV solution is that $p_t = p^*$, as at the intersection point, in each period, t = 1, 2, …. The fiscalist solution, by contrast, implies p_t values given by paths such as that of the thin line in Figure 1. Most of the literature has, however, utilized explicitly optimizing models that imply an analogous diagram as shown in Figure 2, where there is a nonlinear P_t to P_{t+1} mapping that has an positive but increasing slope. Such diagrams are featured, for example, by Carlstrom and Fuerst (2001), Kocherlakota and Phelan (1999), McCallum (2001a), and Christiano and Fitzgerald (2000). Does the E-stability analysis of the foregoing section carry over to specifications such as these?

Although there are some qualifications, the answer is basically "yes." The main point is that E-stability is a local concept, so that conclusions pertaining to the MSV solution in Figure 1 carry over to models of the type in Figure 2. Thus the MSV solution is E-stable. Also, considerable progress toward analysis of non-MSV solutions is reported by Evans and Honkapohja (2001, pp. 267-314). The results are predominately, though not entirely, favorable to the notion that non-MSV solutions are not learnable. In any case, the recent analysis of Evans and Honkapohja (2002b) obtains the same result in a nonlinear model, as mentioned in the preceding footnote.

But other kinds of additional generality are needed, as well. The model that we have used features full price level flexibility and many other simplifications that would not be found in reality. It is of course impossible to generate strict conclusions under assumptions of great generality, but it would appear that the general line of argument provided above will carry over to almost any model that includes a money demand

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this section is that "*the explosive fiscalist price path is unstable under learning*" (2002b, p.7)—i.e., is not LS learnable.

specification of the same general type as (1), where the essential features are that real money demand depends positively upon real transactions and negatively upon an opportunity-cost variable such as R_t that is the difference between the real rates of return on money and other assets. For in any model that includes such a relationship the behavior of the price level will not depart drastically from that of the money stock except along bubble paths—i.e., non-MSV paths. But the literature on E-stability indicates that the type of result found above, that the MSV solution is E-stable and the other solutions are not, obtains in virtually all cases involving well-motivated, plausible models. Evans and Honkapohja (1999, 2001) emphasize some exceptions, but the main examples stem from their (1992) paper. In McCallum (2002), I argue that these examples reflect implausible economic specifications—models that are not "well formulated." It is my impression that there is a very strong association in well formulated models between E-stability and MSV solutions—which implies that MSV solutions are learnable. Non-MSV solutions, by contrast, are typically E-unstable and not learnable by adaptive least-squares procedures.

In a very recent paper, Evans and Honkapohja (2002b) have examined a broader class of policy regimes, following the specification introduced by Leeper (1991). In this case the monetary authority adjusts a one-period nominal interest rate instrument according to a rule of the form

(23) $R_t = \mu_0 + (1 + \mu_1)\Delta p_t + \theta_t$

while simultaneously the fiscal authority is holding $g_t = 0$ and implementing a (lump-sum) tax rule of the form

(24) $tx_t = \tau_0 + \tau_1 b_{t-1} + \zeta_t.$

Leeper (1991) classified monetary policy as "active" if $|(1 + \mu_1)\beta| > 1$ and as "passive" otherwise, and classified fiscal policy as active if $|\beta^{-1} - \tau_1| > 1$ and passive otherwise. Evans and Honkapohja use this terminology, but sensibly focus on cases in which $1 + \mu_1 > 0$ and $\tau_1 > 0$.

Using a linearized version of their model, Evans and Honkapohja (2002b) find that there are two types of solutions that correspond in several ways to the monetarist and fiscalist solutions discussed above. For example, in the monetarist solution the inflation rate depends only upon a constant and θ_t, the current monetary policy shock. For the most part, the monetarist solution is E-stable and LS learnable when monetary policy is active and fiscal policy passive, whereas the fiscalist solution is E-stable and LS learnable when monetary policy is passive and fiscal policy is active. This finding supports the position of Sims (1994) and Leeper (1991) that it is possible for fiscal policy rule settings to influence the behavior of inflation, and in this sense to support claims of the fiscal-theory proponents. But this finding does not overturn the arguments given two paragraphs above. Specifically, when the fiscalist solution is dynamically stable, it involves stable behavior for real money balances and therefore no drastic divergence in the time paths of the money stock and the price level. Such solutions represent cases in which monetary policy is accommodating fiscal policy requirements and therefore represent outcomes that are fully consistent with traditional monetary analysis. There is a small region of the policy parameter space that leads to explosive solutions, which could imply explosive behavior of real money balances and therefore be inconsistent with monetarist doctrines. But these solutions appear to feature explosive behavior of real bond holdings, and thereby to violate transversality conditions that are necessary for private optimality. The relevant region features active monetary and fiscal policy and therefore implies the type of conflicting policy settings discussed above, in which it is appropriate to view fiscal policy in terms of B^S_t settings. In any event, as Evans and Honkapohja (2002b) recognize, there is room for additional analysis of regimes involving the interaction of fiscal and monetary policy rules.

6. Additional Issues

The possibility of price level behavior being dominated by fiscal, rather than monetary, policy stances is the hallmark of the fiscal theory. But there are two additional themes in the literature that deserve brief discussion. The first of these is the occasional appearance of an analysis conducted in the context of a model that does not include any asset with medium of exchange (MOE) properties, i.e., any money. Such discussions have been provided, for example, by Cochrane (1998, 2001), Weil (2002), and Woodford (2002, Ch.2).

The problem with such analyses is that they undermine the very raison d'etre of the fiscal theory. Suppose the model economy has two or more paper assets, one of which is called "money" but has no MOE properties—i.e., does not serve to facilitate transactions in a resource-conserving manner. Then the model pertains to a non-monetary economy in which case there is nothing surprising or interesting about equilibria in which the price level fails to mimic the behavior of this useless and misnamed asset.

Alternatively, suppose that the model recognizes only one paper asset issued by the government that can be thought of either as "bonds" or "money" as, for example, in Weil (2002). In such settings the behavior of the price level, if it is defined as the asset price of goods, will be determined by fiscal policy since the latter governs the creation of that paper asset. But in such a setting there is no distinction between fiscal and monetary policy, so again fiscalist results lose their interest; one could just as well describe management of that asset's supply as monetary policy. In short, a necessary condition for fiscalist results to be of interest is that they occur in models that include both government bonds and MOE money as distinct assets, so that monetary and fiscal policies can conceivably push in different directions.[25]

The second additional theme concerns the viability of an interest-rate pegging policy. Will the price level be well determined in an orthodox model, such as that of Section 2, if the central bank simply keeps the nominal interest rate on bonds fixed at a constant value that implies a zero, or some other, inflation rate? In approaching this issue, note that it can be expressed in terms of a canonical model of the type used extensively in the recent literature on monetary policy rules, e.g., by Clarida, Gali, and Gertler (1999), Woodford (2002), McCallum (2001b), or many others. Such models include an expectational or optimizing "IS" function, a price adjustment relation, and an interest-rate policy rule of the Taylor (1993) type. Typically, such a model might be written as

(24) $y_t = b_0 + b_1(R_t - E_t\Delta p_{t+1}) + E_t y_{t+1} + v_t$

(25) $\Delta p_t = E_t\Delta p_{t+1} + \alpha(y_t - \bar{y}_t) + u_t$

(26) $R_t = \rho + \Delta p_t + \mu_1(\Delta p_t - \pi^*) + \mu_2(y_t - \bar{y}_t)$

[25] Does this argument deny any importance to analyses designed to analyze price level determination in non-monetary economies? In that context, the meaning of the term "price level" becomes an issue. Normally it is the inverse of the exchange value of money, so what is the appropriate definition for an economy without any MOE? It does in fact seem likely that some asset would serve as a common medium of account (MOA) in such an economy, although we have little or no recent experience with such matters, and a government-issued paper asset would be a natural candidate to fill that role. But, in any event, matters of this type need to be explicitly addressed in such analyses.

where $\bar{y}_t$ is the flexible-price (natural rate) value of y_t, presumed exogenous for simplicity, with v_t and u_t being exogenous shock processes. These models would also implicitly include a money-demand function such as (1), but would make no use of it since (24)-(26) is a subsystem that determines the behavior of its three endogenous variables.[26] In the policy rule (26), Δp_t might be replaced by $E_t\Delta p_{t+1}$ or $E_{t-1}\Delta p_{t+j}$ or some other inflation measure. Also, a partial-adjustment version such as

(26') $R_t = (1 - \mu_3)[\rho + \Delta p_t + \mu_1(\Delta p_t - \pi^*) + \mu_2(y_t - \bar{y}_t)] + \mu_3 R_{t-1}$

is often used, to represent interest-rate smoothing behavior by the central bank.

In this literature, it is emphasized that $\mu_1 > 0$ is needed to achieve satisfactory performance; if $\mu_1 \leq 0$ then the "Taylor Principle" is not satisfied.[27] Writers often say that when $\mu_1 \leq 0$ the price level is "indeterminate;" their meaning is that multiple stable RE solutions exist, even when relevant transversality conditions are recognized. The actual problem, as previously argued in McCallum (2001b), is that neither the MSV solution nor the non-MSV bubble solution is E-stable. Thus there is very little prospect of any RE equilibrium being established.

To quickly illustrate that this is the case, let us delete the shocks from (24)-(26) and suppose that prices adjust immediately so that $y_t = \bar{y}_t$. Then the system becomes

(27) $y_t = b_0 + b_1(R_t - E_t\Delta p_{t+1}) + E_t y_{t+1}$

(28) $R_t = \rho + \Delta p_t + \mu_1(\Delta p_t - \pi^*)$.

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[26] Note that, except for the policy rule, our model in Section 2 is a special case in which the shocks are absent and price adjustments are immediate (thereby keeping $y_t = \bar{y}_t$.

[27] Considerable discussion of the Taylor Principle is provided by Woodford (2002), who emphasizes that a smaller value of μ_1 will suffice when $\mu_2 > 0$. In the analysis of Evans and Honkapohja (2002b), incidentally, a slightly different condition is found—but it appears that this is merely the result of a log-

Also let $\overline{y}_t = \overline{y}$ and $\pi^* = 0$. Then we have $0 = b_0 + b_1[\rho + (1 + \mu_1)\,\Delta p_t - E_t\Delta p_{t+1}]$ or

(29) $\Delta p_t = (1 + \mu_1)^{-1} E_t \Delta p_{t+1} + \text{constant}.$

Clearly this system conforms to that of equation (16) above, with $(1 + \mu_1)^{-1} = a_1$. But it has already been demonstrated that for iterative E-stability a necessary condition is $|a_1| < 1$. For unqualified E-stability and least-squares learnability the requirement is, however, $a_1 < 1$. [On this point see Evans (1989) or Evans and Honkapohja (2001).] Thus with $\mu_1 \leq 0$, this condition is not met and learnability does not prevail.

The point of the foregoing with respect to interest rate pegging is, of course, that such a policy is represented by (28) with $\mu_1 = -1$ or $1 + \mu_1 = 0$. Thus the conclusion of E-stability analysis is that pure interest rate pegging—as distinct from use of an activist interest rate instrument—is not a viable policy.

7. Conclusions

Let us conclude with a brief recapitulation of the paper's argument. Basically, it presents a prototype model, in the simplest possible setting, for development and discussion of the fiscal theory of the price level. In this setting, it becomes clear that the fiscal theory's distinctiveness relies upon the analyst's adoption of a bubble solution, rather than the orthodox fundamentals solution, in the context of a multiplicity of rational expectations solutions for the model. It is this step that permits the time path of the price level to depart dramatically from the time path of the money stock, in a model with an orthodox money demand function (as is standard in the literature). To determine which equilibrium is plausible, the paper adopts the criterion of adaptive, least-squares

linear approximation (used commonly but not by them) that serves to eliminates a β term that should appear in the discrete-time version of the Fisher equation.

learnability (since individual agents could not be endowed with perfect knowledge of the economy's true parameter values). By drawing on results developed in the extensive writings of Evans and Honkapohja (1999, 2001, 2002a) it is demonstrated that with the basic policy specification the traditional fundamentals solution is E-stable and therefore learnable, whereas the fiscal-theory bubble solution is not. It is argued that similar results are likely to prevail in more complex models that include the same central ingredients.

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Figure 1

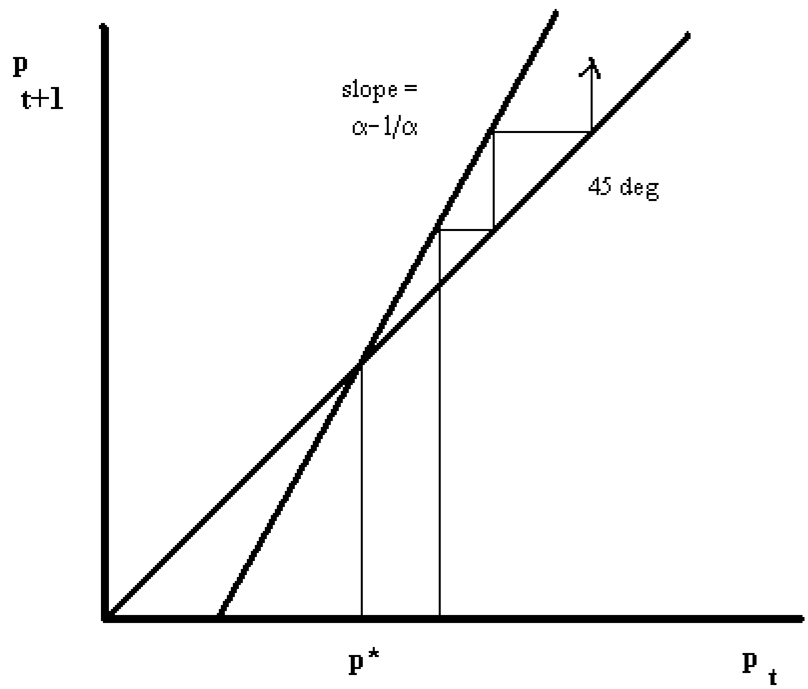

Figure 2